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Dynamic modeling of a cabin pressure control system

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Abstract

Cabin pressure control system of an aircraft maintains cabin pressure in all flight modes as per the aircraft cabin pressurization characteristics by controlling the air flow from the cabin through the outflow valve of the cabin pressure control valve. The movement of outflow valve in turn depends on the air flow from the control chamber of cabin pressure control valve, which is controlled by the clapper and the poppet valves. These valves are actuated by absolute pressure and the differential pressure capsules, respectively depending upon the operating flight conditions. Mathematical models have been developed to simulate the air outflow rates from the cabin and the control chamber of cabin pressure control valve during steady-state and transient flight conditions. These mathematical models have then been translated into a MATLAB program to obtain plots of cabin pressures as a function of aircraft altitudes. The mathematical models are validated for standard cabin pressurization characteristics of a multirole light fighter/trainer aircraft. The model developed, thus can be used to produce a number of variants of cabin pressure control valve to suit different cabin pressurization characteristics.

Keywords

Cabin pressurization system, clapper valve modeling, poppet valve modeling, pressure capsules, pneumatic control of fighter aircraft

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Introduction

In order to ensure smooth flight operation, aircraft environmental control system (AECS)[1] maintain suitable cabin environmental conditions, which are essential for the survival as well as comfort of pilots. The environmental control subsystems include a pneumatic system, air conditioning, cabin pressurization, and supplemental oxygen systems.

The function of cabin pressure control system (CPCS) is to maintain the cabin pressure at a safe level considering physiological requirements, irrespective of altitude at which aircraft fly. Cabin pressure is maintained by regulating the outflow of cold air, served into the cabin from cold air unit.[2] Conditioning air is vented through the cabin pressure control valve (CPCV) that compares the cabin pressure with static ambient pressure and the required pressure inside the cabin.

CPCS is an all pneumatic system which automatically regulates cabin pressure, with no human interference and additional electrical power. It completely works on pneumatic principle, and therefore even the high-intensity electromagnetic field can not affect its performance. It has several other advantages such as light weight, small volume, and low maintenance cost. These characteristics are preferred requirements of modern-day airborne stores.

Pressurization issues

Cabin pressurization is a physiological requirement for pilots and passengers flying in an aircraft. At high altitudes, the partial pressure of oxygen is very low and the body is unable to supply an adequate amount of oxygen through normal breathing to cells and tissues. This lack of oxygen in the blood can cause effects ranging from simple headache and discomfort to total loss of consciousness, depending upon the duration of exposure to the high altitude condition.

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Other effects of low ambient pressure levels are the expansion of gases in cavities (sinus, abdomen) and getting dissolved in the blood (nitrogen) that may cause serious problems.[3]

Middle ear equalization through the Eustachian tube is another physiological problem that happens through pressure changes.[4] The major source of these effects is pressure transients (bumps). Pressure bumps are short duration pressure changes of considerable magnitude to cause pain and discomfort to occupants of the cabin. Prolonged exposure to such pressure bumps can even leads to permanent hearing impairment.[5] Proper design of CPCS can minimize bumps and can enhance pilot comfort. A CPCS must, therefore, maintain adequate cabin pressure levels to ensure the comfort and safety of pilots and passengers.

Existing studies on AECSs have mainly focused on the air conditioning system design and cabin thermal management control. A few studies have focused on the fault diagnosis of the AECS in this regard.[6,7] The need for further study on this area arises due to the fact that despite numerous efforts by various research groups to model cabin pressure regulation system, there is lack of systematic model in open literature that can suit different cabin pressurization profiles. The present work is, thus focused on the study of CPCS through computational simulation of its steady-state and dynamic responses.

Pressurization terms

Environmental control system is a generic term used for system and equipment associated with cooling, heating, ventilation, humidity/contaminant control, and pressurization within passengers and the avionics compartments of aircraft.[8] The following terms should be understood for the discussion of CPCSs:

i. **Aircraft altitude**: The ambient pressure varies with the height from the sea level. Therefore, the pressure at an altitude can be expressed in the form of height from the mean sea level, e.g. ambient pressure of 407 mbar corresponds to an aircraft altitude of 7 km.
ii. **Cabin altitude**: The pressure inside the cabin can be expressed as an aircraft altitude equivalent to the given cabin pressure, e.g. cabin pressure of 752 mbar corresponds to the cabin altitude of 2.44 km.
iii. **Cabin differential pressure**: It is the difference between the cabin and atmospheric pressures.

System components

CPCS follows predefined cabin pressure control laws to limit both the positive and negative cabin differential pressure within the allowable limits with respect to the atmospheric pressure. The system architecture as shown in Figure 1 comprises the following units:

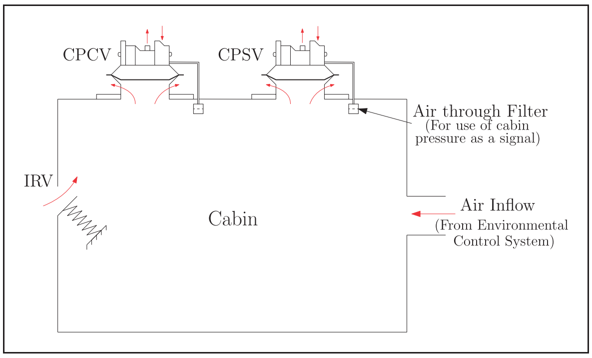

Figure 1. Schematic diagram of cabin pressure control system.

i. Cabin pressure control valve (CPCV)
ii. Cabin pressure safety valve (CPSV)
iii. Inward relief valve (IRV)
iv. Air filter

Cabin pressure control valve

CPCV regulates the cabin pressure according to a given pressurization law as a function of outside atmospheric pressure. The cabin pressure and differential pressure between the cabin and the atmosphere are sensed by an absolute pressure capsule and a differential pressure capsule of the CPCV respectively. The capsules drive two control valve sub-assemblies in the unit to release/build-up air pressure in the control chamber as a function of cabin pressure. The differential pressure between the cabin and control chamber opens or closes the main valve to regulate the outflow of cabin conditioning air to decrease or increase the cabin pressure, respectively.

Cabin pressure safety valve

In the case of failure of CPCV, the CPSV controls the cabin pressure. It maintains the positive pressure differential between the cabin and the outside ambient according to the pressurization law. The general architecture and characteristics of CPSV are similar to CPCV. However, the pressure settings of CPSV are at higher level than CPCV.

Inward relief valve

It limits the negative pressure inside cabin relative to the outside pressure during a rapid descent of aircraft, i.e. when the outside ambient pressure becomes higher than cabin pressure, the valve flap of IRV opens to limit the negative pressure within the pre-defined tolerance values of the pressurization characteristics. The valve consists of a light alloy valve body with a single flap hinged in the shaft and loaded to the closed position by a double helical torsion spring.

Air filter

Air filter ensures the supply of clean air to the control chamber of CPCV and CPSV. The filter is directly connected to the cabin bulkhead union. The unit comprises of an air filter cartridge sandwiched between aluminum alloy body and cover. A wire mesh at the inlet protects the filter cartridge.

System characteristics: Cabin pressurization control law

The CPCS of an aircraft is a fully automatic, pneumatically driven system. On the basis of a proposed cabin pressure control law, it uses the differential pressure between the cabin and external ambient to govern the cabin pressurization at all altitude covered in the flight envelope.

The cabin pressurization control law[9] for a multirole light fighter/trainer aircraft is shown in Figure 2. Taking the structural limitations of the aircraft into consideration, it depicts the way in which cabin altitude (cabin pressure) must vary with aircraft altitude (external ambient pressure) in order to provide appropriate cabin environmental conditions for survival and comfort of pilot and co-pilot based on the physiological requirements.

Normal control law

- From the mean sea level to 2.44 km, the pressure inside the cabin is equal to ambient pressure plus pressure drop due to the flow of air across the CPCV.
- From 2.44 km to 7 km, the cabin pressure remains constant till the differential pressure between cabin and ambient reaches 345 mbar.
- Above 7 km altitude, the CPCS maintains a constant differential pressure of 345 mbar.

Safety control law

In case of failure of the CPCV, the cabin pressure will be regulated by the cabin pressure safety valve according to the following law:

- From the mean sea level to 2.07 km, the pressure inside the cabin is equal to ambient pressure plus pressure drop due to air flow across the CPCV.
- From 2.07 km to 7 km, the cabin pressure remains constant till the differential pressure between cabin and ambient reaches 380 mbar.
- Above 7 km, the CPCS maintains a constant differential pressure of 380 mbar.

Negative pressure relief

The negative pressure relief is ensured by an inward relief valve. It limits the negative differential pressure at a value lower or equal to 20 mbar.

System functioning

The cabin pressure and the differential pressure with respect to external ambient are sensed by an absolute

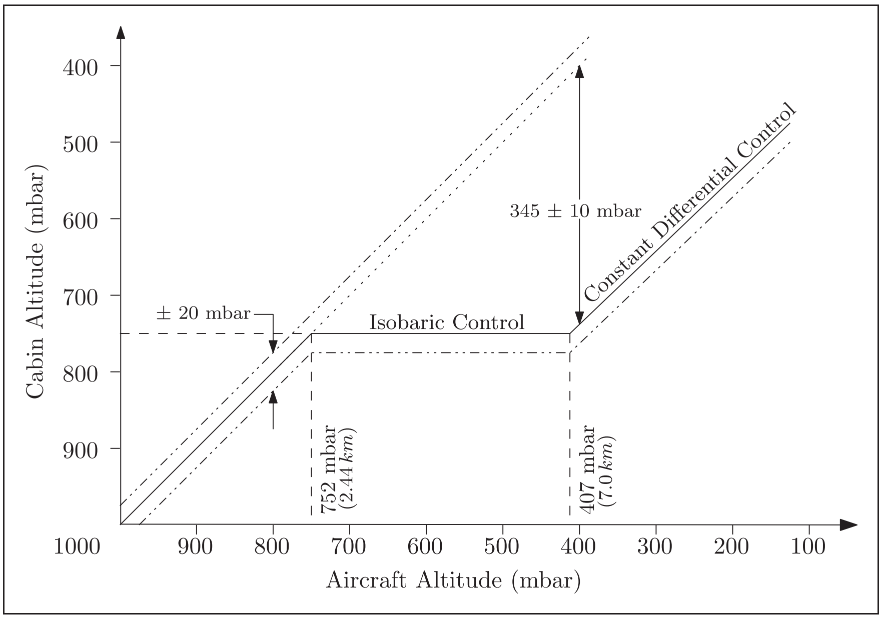

Figure 2. Cabin pressurization control law for a multirole light fighter/trainer aircraft.

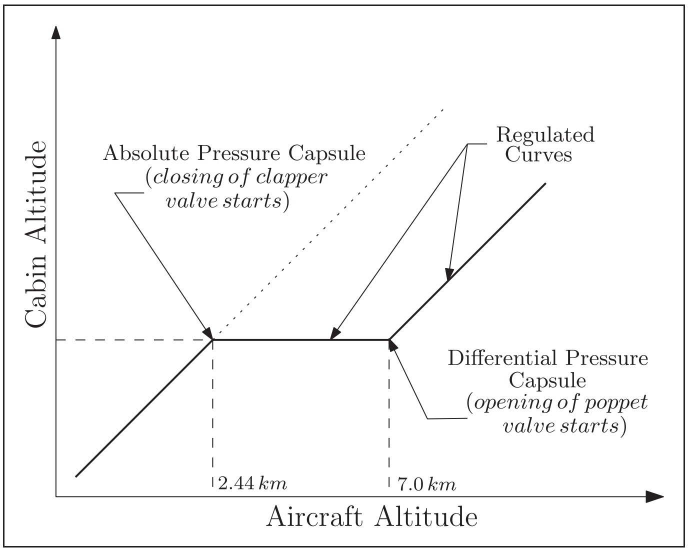

Figure 3. Regulated curve for combined action of absolute and differential pressure capsules.

pressure capsule and a differential pressure capsule respectively (Figure 3). The absolute pressure capsule controls a clapper valve sub-assembly and the differential pressure capsule controls a poppet valve sub-assembly.

The purpose of clapper valve and the poppet valve sub-assemblies is to adjust the outflow from control chamber as a function of the cabin pressure and the differential pressure respectively which in turn generates the driving pressure required for the regulation of outflow from the cabin through the main valve.

i. No pressurization case (0–2.44 km)

Between ground level to an altitude of 2.44 km, the absolute pressure capsule is contracted, and the clapper sub-assembly is fully open on the ground. The differential pressure capsule remains contracted and the poppet valve is closed. The control chamber is fed cabin air through a nozzle (jet) whose passage area is less than that of open clapper valve sub-assembly as shown in Figure 4. This allows the control chamber pressure to be kept almost equal to the outside pressure. The main valve opens to allow the outflow of conditioning air and keeps the cabin pressure equal to the external ambient pressure plus the pressure drop across the main valve.

ii. Constant cabin pressure case (2.44–7 km)
At level flight altitude, i.e. from 2.44 km to 7 km, the absolute pressure capsule expansion causes the clapper valve to close such that the clapper valve allows air flow, which is less than the jet flow. The differential pressure capsule remains contracted and the poppet valve is closed. The absolute pressure capsule expands to control the pressure in the control chamber. The main valve assumes a balanced position corresponding to the pressure in the control chamber and cabin pressure remains constant between 2.44 km to 7 km altitude level.

iii. Constant differential pressure case (7 km and above)

Above 7 km, when the differential pressure between the cabin and ambient reaches a preset value, the differential pressure capsule expansion causes the poppet valve sub-assembly to start opening. At this stage, the clapper valve sub-assembly becomes fully closed. The increase in airflow from the control chamber to the external ambient reduces the control chamber pressure, and therefore further opens the main valve.

Mathematical model

Continuity equation for control volume (cabin)

Applying the principle of continuity[10] to the cabin as a control volume shown in Figure 4, the rate of change of the accumulated mass inside the cabin can be expressed by

$$\dot{m}_c = \dot{m}_{in} - \dot{m}_{out} - \dot{m}_{leak} - \dot{m}_{jet} \tag{1}$$

where $\dot{m}_c$ is the rate of change of air-mass within the cabin, $\dot{m}_{in}$ is the mass flow rate of air coming into the cabin, $\dot{m}_{out}$ is the mass flow rate of air from cabin to atmosphere through CPCV, whereas $\dot{m}_{jet}$ is the mass flow rate of air from cabin to control chamber through jet and $\dot{m}_{leak}$ is the air mass leakage rate from the valve assembly.

Under steady-state condition, $\dot{m}_c = 0$. Therefore, the continuity equation (1) gets modified as

$$\dot{m}_{out} = \dot{m}_{in} - \dot{m}_{leak} - \dot{m}_{jet} \tag{2}$$

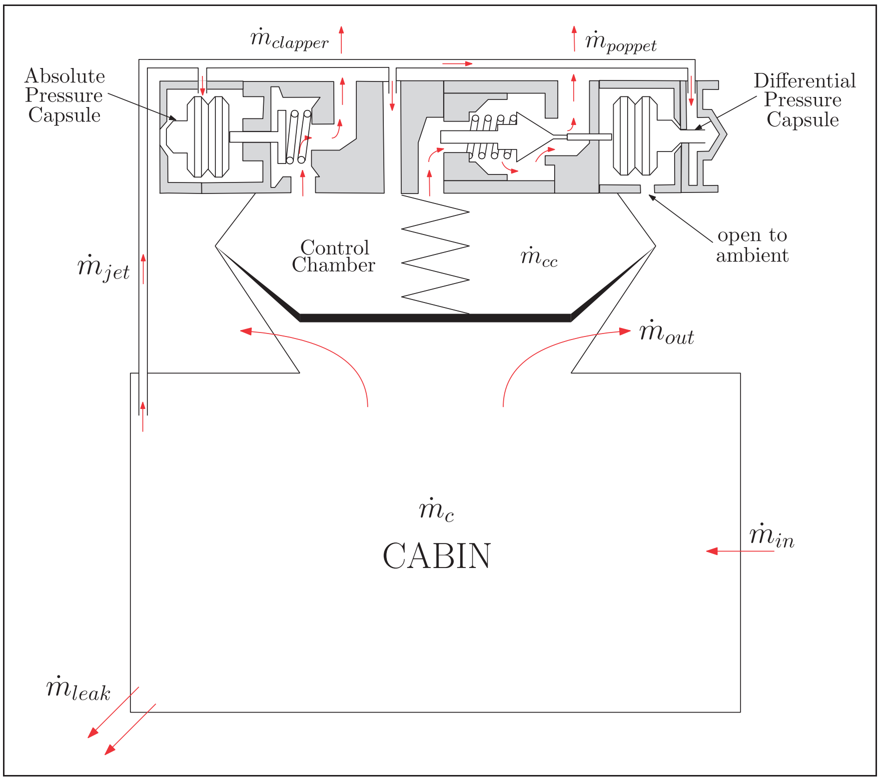

Figure 4. Schematic diagram of CPCS showing the system components along with the air mass flow direction in/out of the system.

Considering the compressible airflow through the outflow valve, the following equation is obtained from isentropic flow theory[11] to determine the mass outflow rate from the cabin to the atmosphere. Refer to Appendix I (Isentropic flow of a gas through a nozzle)

$$\dot{m}_{out} = C_d A_{valve} \sqrt{\frac{2\gamma}{\gamma-1} P_c \rho_c \left[\left(\frac{P_a}{P_c}\right)^{\frac{2}{\gamma}} - \left(\frac{P_a}{P_c}\right)^{\frac{\gamma+1}{\gamma}} \right]} \tag{3}$$

where P_a and P_c are the pressures corresponding to ambient and cabin respectively, γ is the isentropic index, ρ_c is the density of air inside the cabin, A_{valve} is the opening area of the main valve, and C_d is the coefficient of discharge for flow through the main valve.

Similarly, using the concept of compressible and isentropic flow, the amount of air mass flow rate out of the cabin to control chamber through the jet is governed by the following equation

$$\dot{m}_{jet} = C'_d A_{jet} \sqrt{\frac{2\gamma}{\gamma-1} P_c \rho_c \left[\left(\frac{P_{cc}}{P_c}\right)^{\frac{2}{\gamma}} - \left(\frac{P_{cc}}{P_c}\right)^{\frac{\gamma+1}{\gamma}} \right]} \tag{4}$$

where $A_{jet} = (\pi d_{jet}^2/4)$, P_{cc} is the control chamber pressure, C'_d is the coefficient of discharge for flow through jet, whereas d_{jet} and A_{jet} are the diameter and area of the jet, respectively.

Formulation of an opening area of the outflow valve

In order to attain the required airflow through the outflow valve, the opening area of the valve must be modulated.[12] Using equation (3), the following equation is obtained to determine the required opening area

$$A_{valve} = \frac{\dot{m}_{out}}{C_d \sqrt{\frac{2\gamma}{\gamma-1} P_c \rho_c \left[\left(\frac{P_a}{P_c}\right)^{\frac{2}{\gamma}} - \left(\frac{P_a}{P_c}\right)^{\frac{\gamma+1}{\gamma}} \right]}} \tag{5}$$

Using a cylindrical surface area as shown in Figure 5 for representing the geometry of the outflow valve, the main spring deflection required to maintain the opening area is given by

$$y = \frac{A_{valve}}{2\pi r} \tag{6}$$

where y is the vertical lift of the main membrane/diaphragm, whereas r and R are the inner and outer radii of the diaphragm, respectively.

Governing equation of motion for the diaphragm

The fluid force, F_{fm}, acting on the membrane is given by

$$F_{fm} = (P_c - P_a)\pi r^2 - (P_{cc} - P_a)\pi R^2 \tag{7}$$

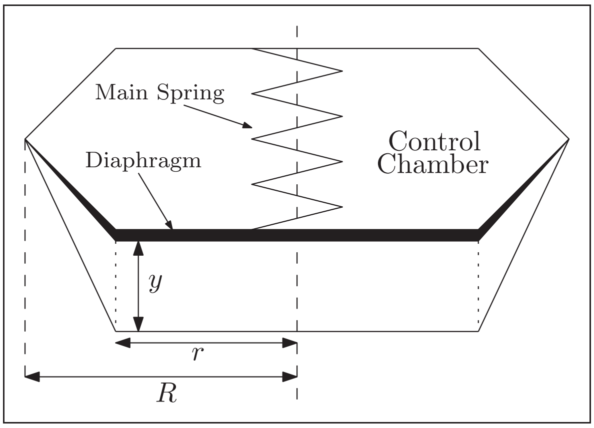

Figure 5. Schematic diagram of control chamber with inner and outer radiuses of cylindrical opening area.

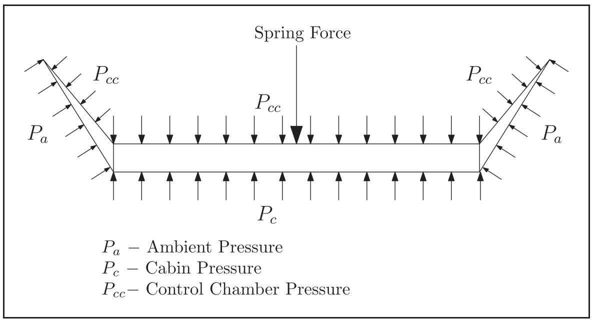

Figure 6. Air pressure and spring force acting on the diaphragm.

Once the lift in the main valve is known, and hence the spring force, the net force acting on the diaphragm, F_m, can be obtained using the following equation

$$F_m = (P_c - P_a)\pi r^2 - (P_{cc} - P_a)\pi R^2 - k_{sp}y \tag{8}$$

where k_{sp} is the stiffness value of the spring attached to the diaphragm.

Applying force equilibrium on the diaphragm as shown in Figure 6, the following relationships are obtained for the two cases:

- if y increases

$$m_{dph}\ddot{y} + c_{dph}\dot{y} + k_{sp}y = (P_c - P_a)\pi r^2 - (P_{cc} - P_a)\pi R^2 \tag{9}$$

- if y decreases

$$-m_{dph}\ddot{y} - c_{dph}\dot{y} + k_{sp}y = (P_c - P_a)\pi r^2 - (P_{cc} - P_a)\pi R^2 \tag{10}$$

where m_{dph} and c_{dph} are the mass and damping coefficient of the diaphragm system, respectively.

Continuity equation for control chamber

Considering the control chamber as a control volume as shown in Figure 4 and applying the principle of continuity, the following continuity equations are obtained

$$\dot{m}_{cc} = \dot{m}_{jet} - \dot{m}_{valve} \tag{11}$$

$$\dot{m}_{valve} = \dot{m}_{clapper} + \dot{m}_{poppet} \tag{12}$$

where $\dot{m}_{cc}$ is the rate of change of air-mass within the control chamber, $\dot{m}_{clapper}$ and $\dot{m}_{poppet}$ are the mass flow rates of air from control chamber to atmosphere through clapper and poppet valves respectively, whereas $\dot{m}_{valve}$ is the combined mass flow rate of air from control chamber to atmosphere.

For proper application of valve, $\dot{m}_{valve}$ should have the following behavior with altitude, h

$$\dot{m}_{valve} = \begin{cases} \dot{m}_{clapper}, & 0 \leqslant h \leqslant 7.0\,\text{km} \\ \dot{m}_{poppet}, & h > 7.0\,\text{km} \end{cases} \tag{13}$$

(i) No pressurization Case (0–2.44 km)

The air mass flow rate going out of the control chamber to the ambient is

$$\dot{m}_{clapper} = C'_d A_{seat} \sqrt{\frac{2\gamma}{\gamma-1} P_{cc}\rho_{cc}\left[\left(\frac{P_a}{P_{cc}}\right)^{\frac{2}{\gamma}} - \left(\frac{P_a}{P_{cc}}\right)^{\frac{\gamma+1}{\gamma}}\right]} \tag{14}$$

During this altitude range, the area at the valve seat, as shown in Figure 7, is more than the annulus area between the clapper valve and the valve seat, therefore

$$A_{seat} = \frac{\pi d_2^2}{4} \tag{15}$$

where d_2 and A_{seat} are the diameter and cross-sectional area of clapper valve seat opening respectively, whereas ρ_{cc} is the density of air inside the control chamber and C'_d is the coefficient of discharge for flow through clapper valve.

(ii) Constant cabin pressure case (2.44–7 km)

In this altitude range, the flow passes only through clapper valve sub-assembly and is given by

$$\dot{m}_{clapper} = C'_d A_{clapper} \sqrt{\frac{2\gamma}{\gamma-1} P_{cc}\rho_{cc}\left[\left(\frac{P_a}{P_{cc}}\right)^{\frac{2}{\gamma}} - \left(\frac{P_a}{P_{cc}}\right)^{\frac{\gamma+1}{\gamma}}\right]} \tag{16}$$

Since the absolute pressure capsule is used to control the pressure in the control chamber, it does so by controlling the effective flow area of the clapper valve shown in Figure 7, i.e. the circumferential area around the clapper valve, which would be less than the valve seat area, and therefore governs the outflow rate from control chamber to ambient through clapper valve sub-assembly and is given by the following equation

$$A_{clapper} = \pi d_2 x_{clapper} \tag{17}$$

where $A_{clapper}$ is the circumferential area around the clapper valve and its valve seat sub-assembly and $x_{clapper}$ is the gap between clapper valve and its valve seat.

The absolute pressure capsule chamber senses the cabin pressure only, hence, its displacement with respect to the variation in the cabin pressure, P_c, is characterized by following quintic polynomial equation[a]

$$x_{abs} = p_1 P_c^5 + p_2 P_c^4 + p_3 P_c^3 + p_4 P_c^2 + p_5 P_c + p_6 \tag{18}$$

where x_{abs} is the displacement of the absolute pressure capsule and p_{1-6} are polynomial constants.

The motion of the clapper valve, $x_{clapper}$, is given by the following equation with $P_c^{ref} = P_c|_{2.44\,\text{km}}$

$$m_{clp}\ddot{x}_{clapper} + c_{clp}\dot{x}_{clapper} + k_{clp}x_{clapper} = k_{clp}\left(x_{abs}|_{P_c} - x_{abs}|_{P_c^{ref}}\right) \tag{19}$$

where m_{clp} and c_{clp} are the mass and damping coefficient of the clapper valve respectively and k_{clp} is the stiffness value of the clapper-spring.

Under steady-state condition $x_{clapper} = x_{abs}|_{P_c} - x_{abs}|_{P_c^{ref}}$ and at $h = 2.44$ km, $x_{clapper} = 0$

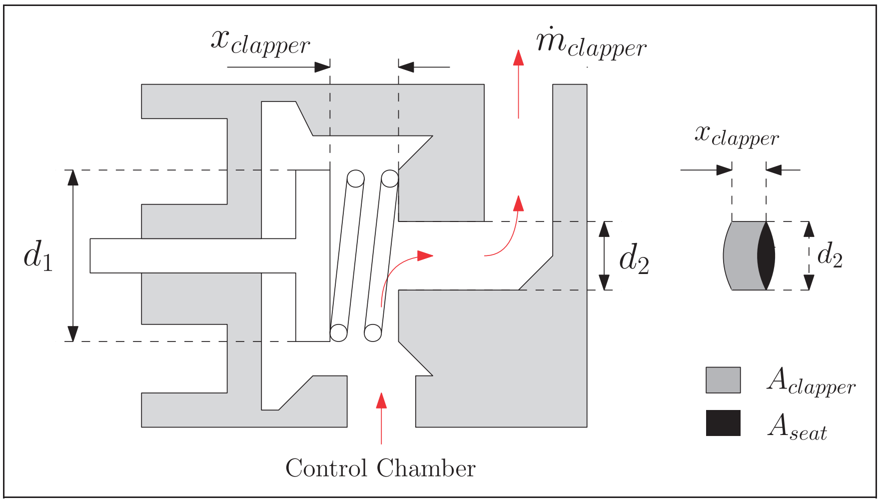

Figure 7. Clapper valve sub-assembly.

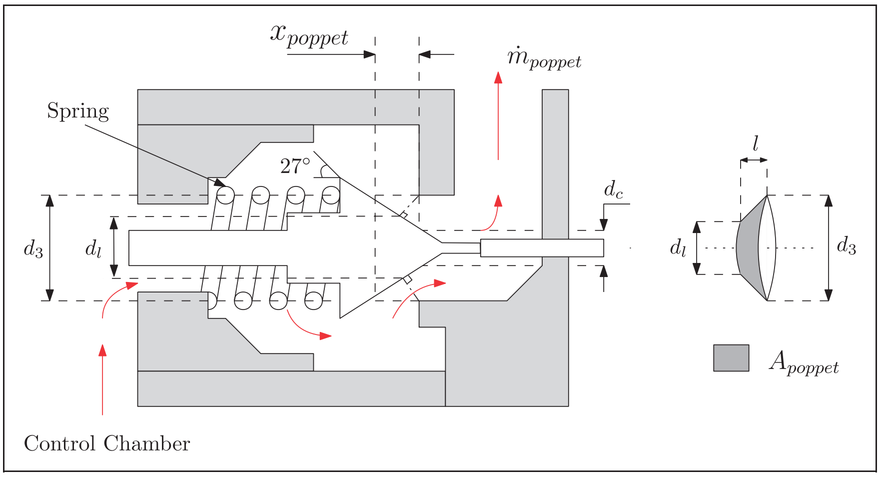

Figure 8. Poppet valve sub-assembly.

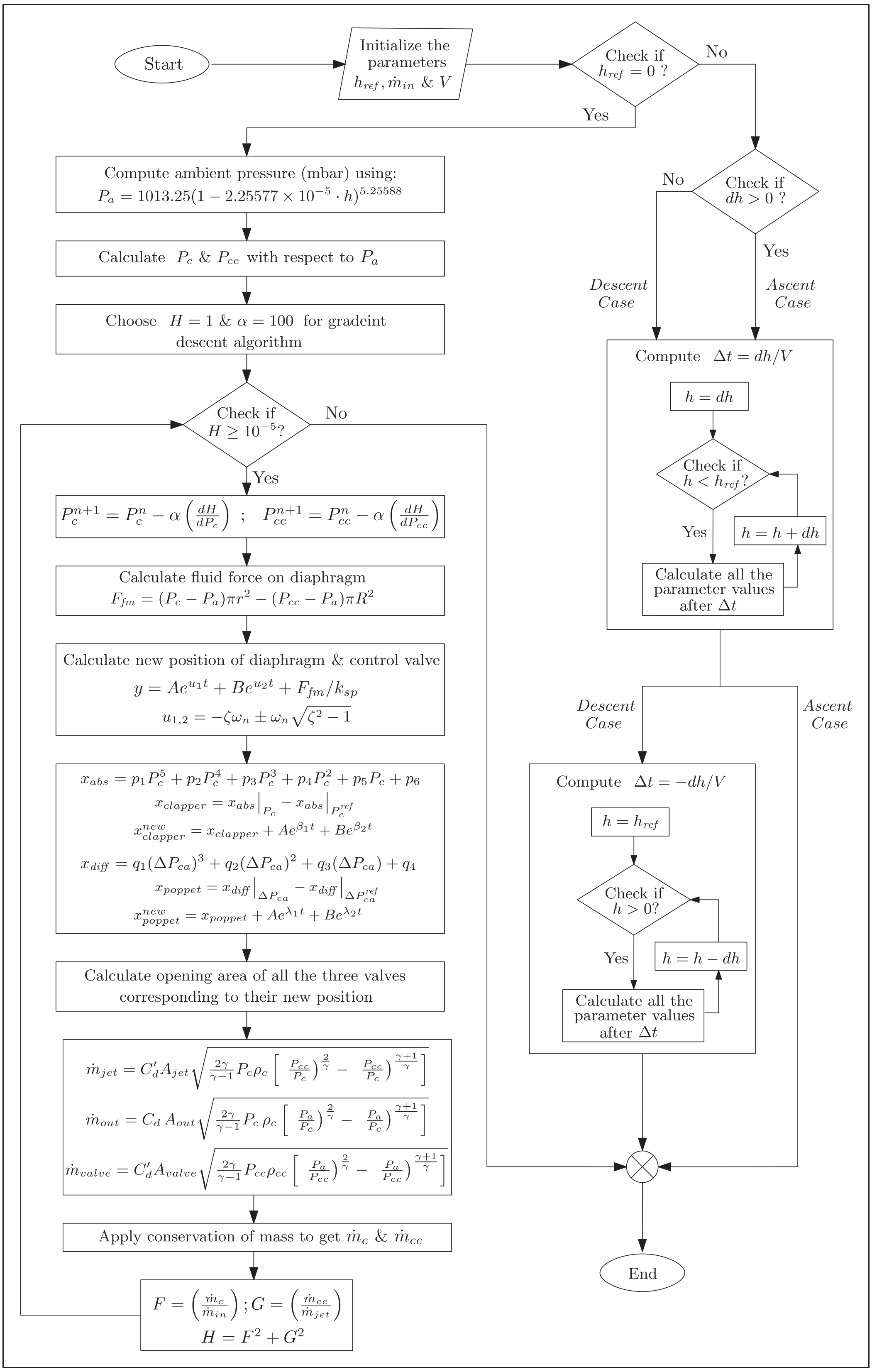

Figure 9. Flowchart diagram of the combined algorithm for all the pressurization cases.

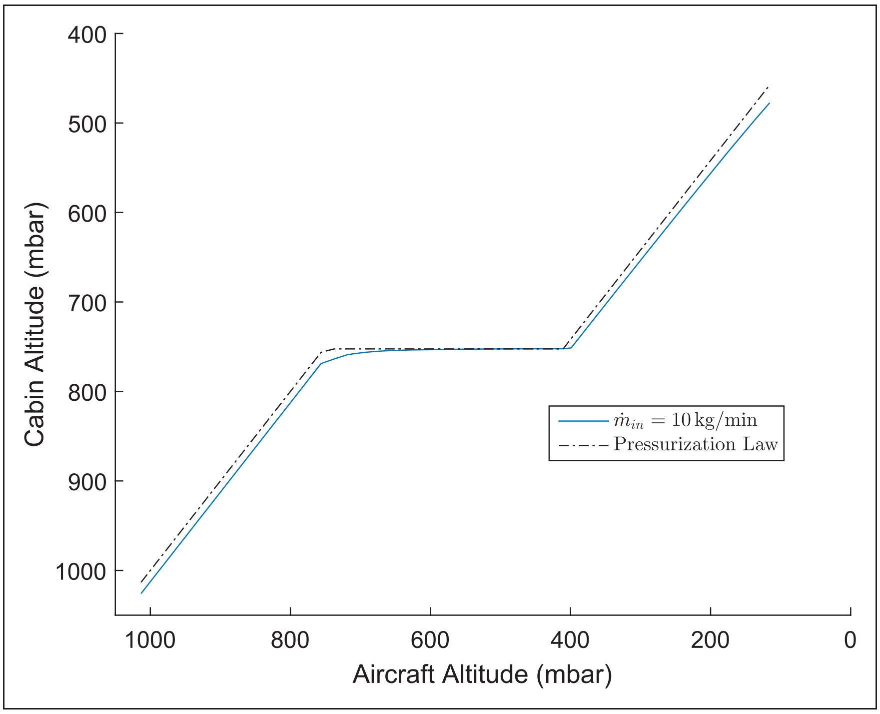

Figure 10. Cabin pressure output inline with the cabin pressurization law for mass inflow rate ($\dot{m}_{in} = 10$ kg/min).

(iii) Constant differential pressure case (7 km and above)

For this altitude range, the flow through the poppet valve is governed by the following equation

$$\dot{m}_{poppet} = C'_d A_{poppet} \sqrt{\frac{2\gamma}{\gamma-1} P_{cc}\rho_{cc}\left[\left(\frac{P_a}{P_{cc}}\right)^{\frac{2}{\gamma}} - \left(\frac{P_a}{P_{cc}}\right)^{\frac{\gamma+1}{\gamma}}\right]} \tag{20}$$

where A_{poppet} is the cross-sectional area at the poppet valve seat sub-assembly.

In this case, the control-chamber pressure is completely regulated by the differential pressure capsule alone, and it does so by controlling the effective flow area of the poppet valve as shown in Figure 8, i.e. the cross-sectional area at the poppet valve seat, A_{poppet}, is given by the following equation

$$\begin{aligned} A_{poppet} &= \frac{\pi}{4}(d_3 + d_l)(d_3 - d_c)\cos(27^\circ) \\ \text{where, } d_c &= d_3 - 2x_{poppet}\tan(27^\circ) \\ d_l &= d_3 - 2(x_{poppet} - l)\tan(27^\circ) \\ l &= \frac{1}{2}(d_3 - d_c)\sin(27^\circ)\cos(27^\circ) \end{aligned} \tag{21}$$

whereas x_{poppet} is the gap between the poppet valve and its valve seat, d_3 is the diameter of poppet valve seat opening, and d_c is the diameter of part of the poppet cone which restricts the flow through poppet valve seat opening.

Since the differential pressure capsule senses the difference between cabin and ambient pressure, its traversal with respect to the differential pressure is characterized by the following cubic polynomial equation

$$x_{diff} = q_1(\Delta P_{ca})^3 + q_2(\Delta P_{ca})^2 + q_3(\Delta P_{ca}) + q_4 \tag{22}$$

where $\Delta P_{ca} = P_c - P_a$, x_{diff} is the displacement of the differential pressure capsule and q_{1-4} are polynomial constants.

The motion of the poppet valve, x_{poppet}, is given by the following equation with $\Delta P_{ca}^{ref} = P_c|_{2.44\,\text{km}} - P_a|_{7.0\,\text{km}}$

$$\begin{aligned} m_{pop}\ddot{x}_{poppet} + c_{pop}\dot{x}_{poppet} + k_{pop}x_{poppet} \\ = k_{pop}\left(x_{diff}|_{\Delta P_{ca}} - x_{diff}|_{\Delta P_{ca}^{ref}}\right) \end{aligned} \tag{23}$$

where m_{pop} and c_{pop} are the mass and damping coefficient of the poppet valve respectively and k_{pop} is the stiffness value of the poppet-spring.

Under steady-state condition $x_{poppet} = x_{diff}|_{\Delta P_{ca}} - x_{diff}|_{\Delta P_{ca}^{ref}}$ and at $h = 7.0$ km, $x_{poppet} = 0$.

Choked flow in the valves

The maximum differential pressure (ΔP_{ca}) maintained in the cabin is corresponding to 345 mbar.

For choking in the main valve, the critical pressure ratio is given by

$$\frac{P_a}{P_c} = \left(\frac{2}{\gamma+1}\right)^{\gamma/\gamma-1} \quad (24)$$

For air, $\gamma = 1.4$

$$\frac{P_a}{P_c} = 0.528$$

$$\frac{P_a}{P_a + 345} = 0.528 \quad (25)$$

The value of ambient pressure obtained using equation (25) is 385.93 mbar, which corresponds to 7.4 km altitude. Thus, the main valve is choked beyond this altitude and the control chamber pressure follows cabin pressure variation. The poppet valve will, therefore, be also choked but at an altitude

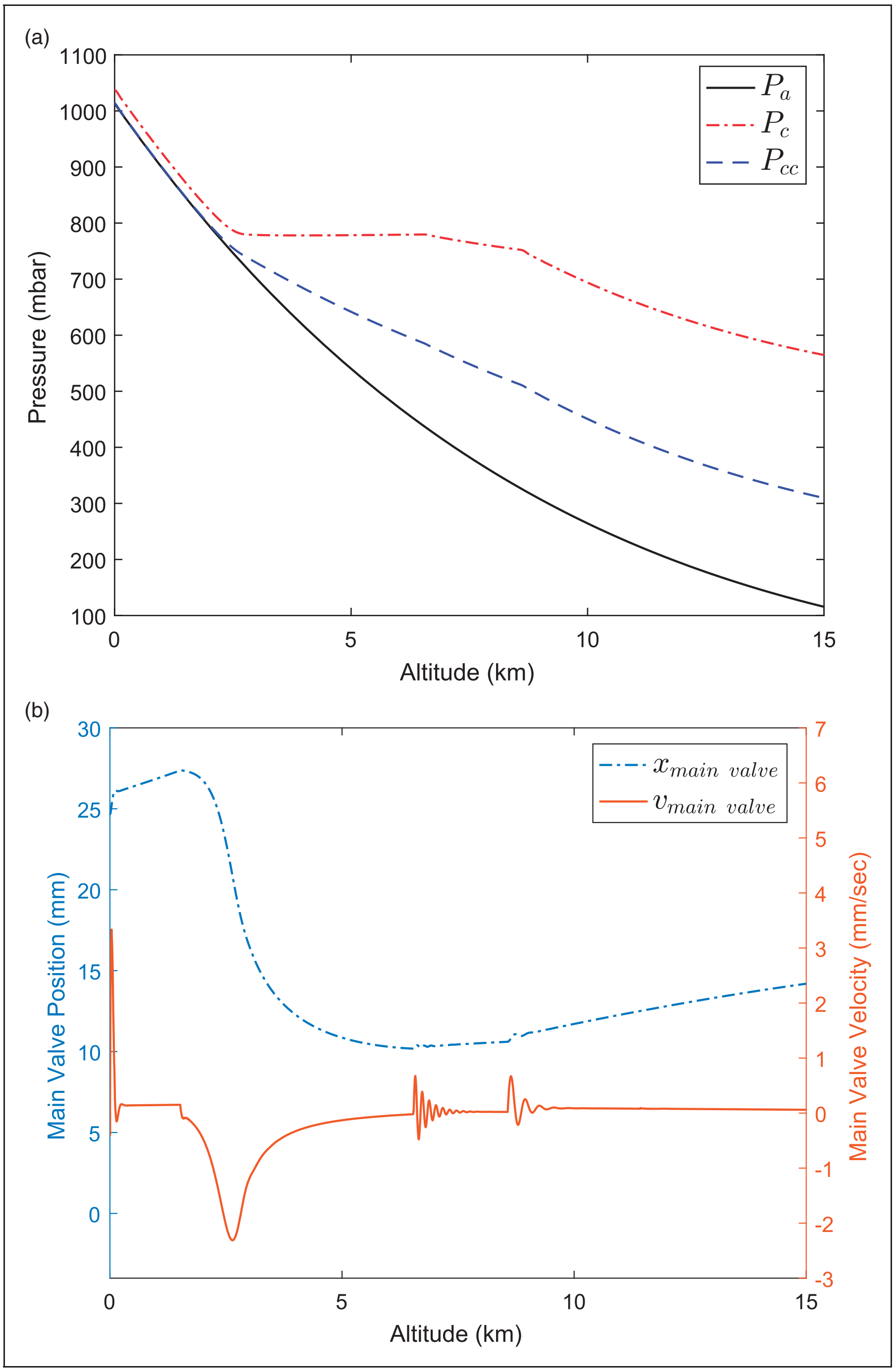

Figure 11. Case I: Air mass flow rate to cabin ($\dot{m}_{in} = 25$ kg/min), aircraft climb rate ($V = 150$ m/s): (a) air pressure vs. altitude; (b) main valve position and velocity vs. altitude.

slightly greater than 7.4 km since the control chamber pressure is less than the cabin pressure.

Design methodology

The algorithm developed for the steady-state condition for the CPCS is shown in Figure 9.

Gradient descent algorithm

For finding steady-state solution, there should not be any accumulation of mass inside the cabin and control chamber (i.e. $\dot{m}_c = 0$ and $\dot{m}_{cc} = 0$). This can be achieved using gradient descent algorithm.

Gradient descent is a first-order iterative optimization algorithm, which is used to find the minimum value of a function. If H is a function in two variables, such that $H = f(P_c, P_{cc})$

$$H = \left(\frac{\dot{m}_c}{\dot{m}_{in}}\right)^2 + \left(\frac{\dot{m}_{cc}}{\dot{m}_{jet}}\right)^2 \tag{26}$$

then the minimum value of H occurs in the direction which is proportional to the negative of the gradient of H at that point.

$$\begin{aligned} P_c^{n+1} &= P_c^n - \alpha \left.\frac{dH}{dP_c}\right|_{P_c^n} \\ P_{cc}^{n+1} &= P_{cc}^n - \alpha \left.\frac{dH}{dP_{cc}}\right|_{P_{cc}^n} \end{aligned} \tag{27}$$

where α is the constant of proportionality in gradient descent algorithm.

Spring-mass-damper system

The solution of the differential equations (9) and (10) for spring mass damper system can be given as

$$y = Ae^{u_1 t} + Be^{u_2 t} + F_{fm}/k_{sp} \tag{28}$$

The roots of above differential equation are as follows: $u_{1,2} = -\zeta\omega_n \pm \omega_n\sqrt{\zeta^2 - 1}$ where ζ and ω_n are the damping coefficient and natural frequency of the spring-mass-damper system, respectively.

Results and discussions

The algorithms developed for the steady-state and the transient conditions have been put in the form of flowchart as shown in Figure 9 for developing the MATLAB code.

The MATLAB code developed was run for the following flight cases to obtain cabin pressure variations:

- **Case I**: Mass inflow rate ($\dot{m}_{in} = 25$ kg/min), aircraft climb rate ($V = 150$ m/s) Refer Figure 11(a) and (b).
- **Case II**: At constant aircraft altitude ($h = 6.0$ km), Mass inflow rate variation ($\dot{m}_{in} = 18-2$ kg/min) Refer Figure 12(a) and (b).
- **Case III**: At constant aircraft altitude ($h = 6.0$ km), Mass inflow rate variation ($\dot{m}_{in} = 2-18$ kg/min) Refer Figure 13(a) and (b).

Cabin pressure is plotted first as a function of altitude for Case I and as time-function for Cases II and III and variations are shown in Figures 11(a), 12(a), and 13(a). Similarly, main valve travel, which largely determines the cabin pressure variations, has also been plotted first as a function of aircraft altitude for Case I and as time-function for Cases II and III.

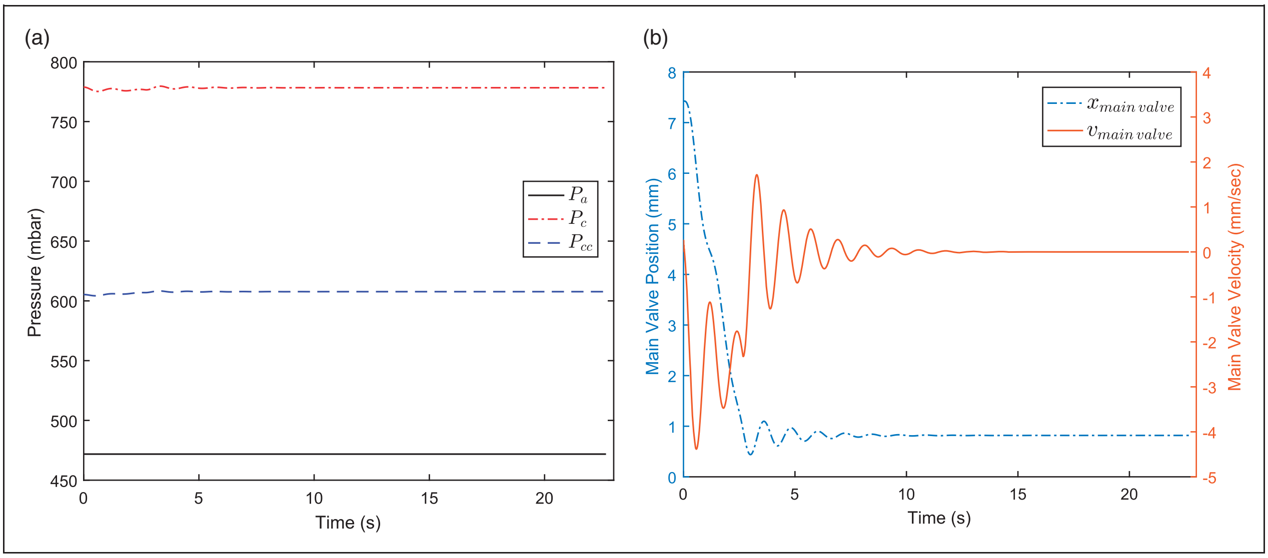

Figure 12. Case II: At constant aircraft altitude ($h = 6.0$ km), air mass flow rate variation to cabin ($\dot{m}_{in} = 18 - 2$ kg/min): (a) air pressure vs. time; (b) main valve position and velocity vs. time.

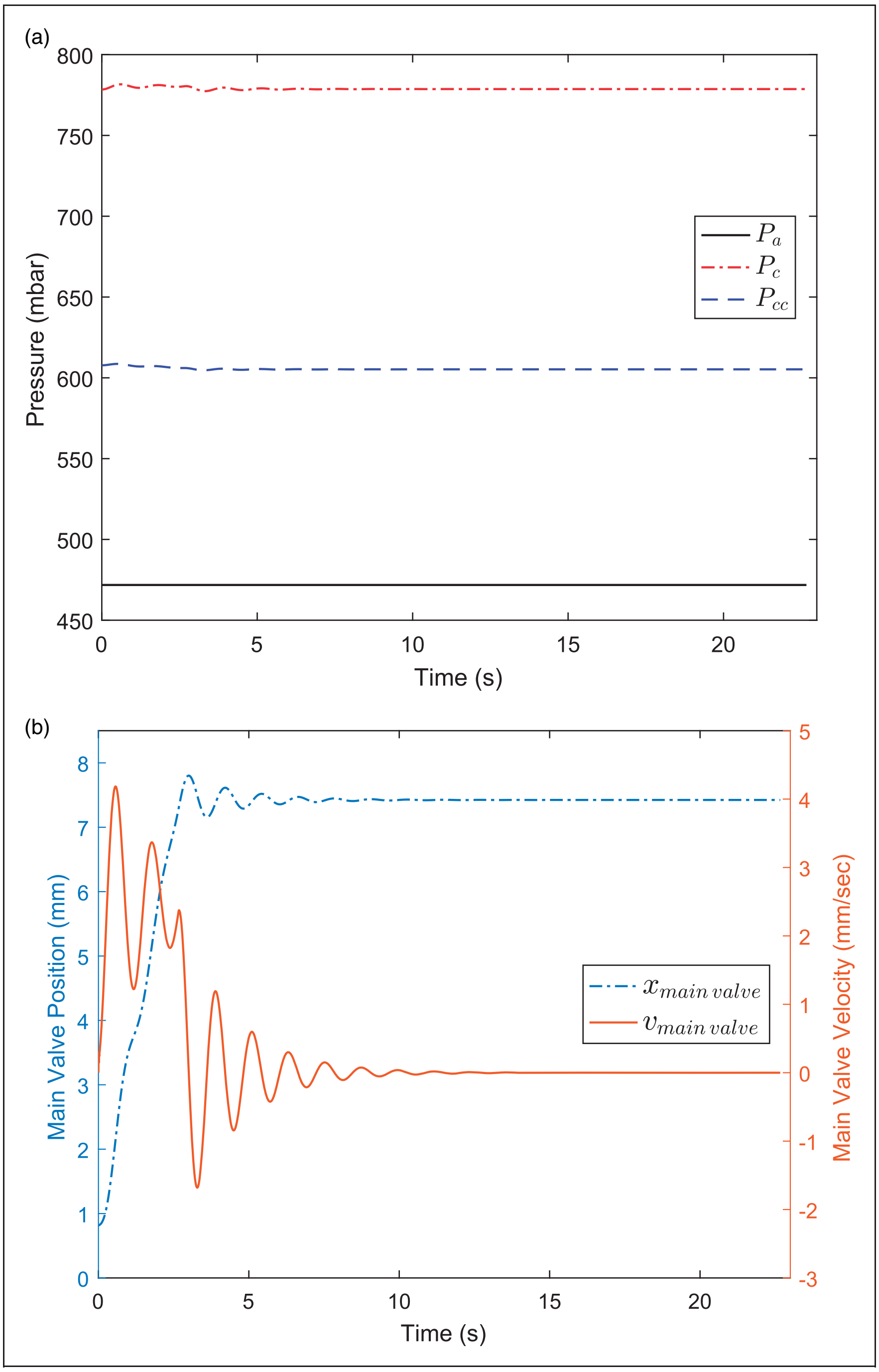

Figure 13. Case III: At constant aircraft altitude ($h = 6.0$ km), air mass flow rate variation to cabin ($\dot{m}_{in} = 2-18$ kg/min): (a) air pressure vs. time; (b) main valve position and velocity vs. time.

The variations are shown in Figures 11(b), 12(b), and 13(b).

Figures 10 and 11(a) show the cabin pressure variations which is inline with the desired pressurization characteristics (Figure 2) of aircraft cabin for different flight cases. The main valve displacement and velocity plot of Figure 11(b) shows small perturbations and this is reflected in the cabin pressure variation of Figure 11(a) as well. However, these oscillations are small and within the acceptable value of ± 20 mbar. At constant aircraft altitude, the cabin pressure variations of Figures 12(a) and 13(a) continue to be within the tolerance band when the air mass flow rate varies. However, the air mass flow variations into the cabin for both cases of increasing and decreasing flow have oscillations in displacement and velocity of the main valve (Figures 12(b) and 13(b)). The cabin pressure variations still remain within the acceptable range. This is because the variation in the magnitude of air mass flow going out of the cabin through the main valve is significantly small as also the associated time-period compared to a large volume of the cabin to make any perceptible change in cabin pressure.

Conclusion

Mathematical models have been developed for design and performance analysis of CPCS of a multirole light fighter/trainer aircraft. The mathematical models cover steady-state and transient flight conditions for the control volume depicting aircraft cabin and relate to a typical CPCV architecture used for determining a given aircraft cabin pressurization characteristics. A competent MATLAB program has been developed in reference to the mathematical models for various elements of CPCV.

Cabin pressure plots obtained from the MATLAB program for steady-state and transient flight conditions lie within the tolerance band of the cabin pressurization characteristics curve of the aircraft for multiple simulations in different flight cases. This validates the correctness and robustness of the mathematical models developed. The MATLAB code developed can, therefore, be used to produce number of CPCV designs inline with the desired cabin pressurization characteristics through controlling the flow passage geometry of the jet, the clapper valve and the poppet valve. In future, the extension of this CPCS with active control system will be presented, which will provide robustness and easy adaptability to variable pressurization profile by augmenting a series of smart actuators.

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Note

a. Goodness of fit: SSE = 3.249e-34 and R-square = 1. The sum of squares due to error (SSE) measures the total deviation of the response values from the fit to the response values. A value closer to 0 indicates that the model has a smaller random error and that the fit will be more useful for prediction whereas, R-square is the square of the correlation between the response values and the predicted response values. R-square can take on any value between 0 and 1, where a value closer to 1 indicates that a greater proportion of variance is accounted for by the model.

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Appendix 1

Isentropic flow of a gas through a nozzle

Consider the flow in a convergent-divergent nozzle as shown in Figure 14. From the first law of thermodynamics,[13] the energy equation is written as:

$$\bar{h}_1 + \frac{v_1^2}{2} = \bar{h}_2 + \frac{v_2^2}{2} = \bar{h}_0 \tag{29}$$

$$v_2 \approx \sqrt{2(\bar{h}_1 - \bar{h}_2)}; \quad \text{for} \quad (v_2 \gg v_1) \tag{30}$$

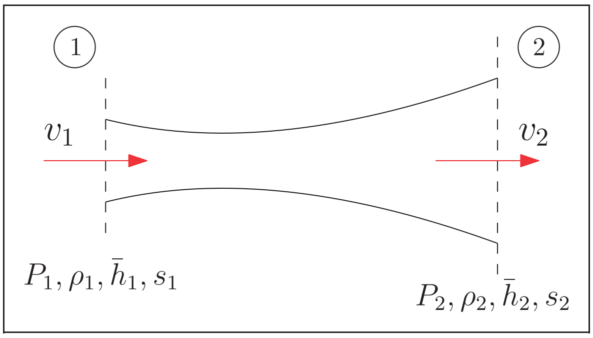

Figure 14. Gas flowing through a converging–diverging nozzle.

where $\bar{h}_0$ is the stagnation enthalpy, $(\bar{h}_1 - \bar{h}_2)$ is the enthalpy drop across the nozzle, and v is the throat velocity.

From the second law of thermodynamics

$$s_2 - s_1 \geqslant 0 \tag{31}$$

Now the entropy change is calculated using

$$Tds = d\bar{h} - vdP \tag{32}$$

where $\bar{h}$ is the specific enthalpy, s is the specific entropy, T is the temperature, and v is the specific volume.

For the isentropic flow, $Tds = 0$

$$d\bar{h} = vdP = \frac{dp}{\rho} \tag{33}$$

Upon integrating equation (33) over stages 1 and 2

$$\begin{aligned} \int_1^2 \mathrm{d}\bar{h} &= \int_1^2 \frac{\mathrm{d}P}{\rho} \\ (\bar{h}_1 - \bar{h}_2) &= \int_1^2 \frac{\mathrm{d}P}{\rho} \end{aligned} \tag{34}$$

Assuming that the pressure and volume of the gas obey the isentropic gas law during the process, the governing equation is given by

$$\frac{P_1}{\rho_1^\gamma} = \frac{P_2}{\rho_2^\gamma} = const. \tag{35}$$

where γ is the isentropic index.

Further integrating equation (34) and applying isentropic law equation (35)

$$\begin{aligned} (\bar{h}_1 - \bar{h}_2) &= \int_1^2 \frac{\mathrm{d}P}{\rho} \\ &= -\frac{\gamma}{\gamma - 1}\left[\left(\frac{P_2}{\rho_2^\gamma}\right)\left(P_2^{\frac{\gamma}{\gamma-1}} - P_1^{\frac{\gamma}{\gamma-1}}\right)\right] \\ &= \frac{\gamma}{\gamma-1}\left(\frac{P_2}{\rho_2} - \frac{P_1}{\rho_1}\right) \end{aligned}$$

$$(\bar{h}_1 - \bar{h}_2) = \frac{\gamma}{\gamma-1}\left(\frac{P_2}{\rho_2} - \frac{P_1}{\rho_1}\right) \tag{36}$$

Now the mass flow rate is given by

$$\dot{m} = \rho_2 A_2 v_2 \tag{37}$$

Combining equations (30), (35), (36), and (37) to obtain the mass flow rate at the exit of the nozzle having a coefficient of discharge, C_d, as follows

$$\dot{m} = C_d A_2 \sqrt{\frac{2\gamma}{\gamma-1} P_1 \rho_1 \left[\left(\frac{P_2}{P_1}\right)^{\frac{2}{\gamma}} - \left(\frac{P_2}{P_1}\right)^{\frac{\gamma+1}{\gamma}}\right]} \tag{38}$$

Appendix

Notation

$A_{clapper}$	circumferential area around the clapper valve and its valve seat sub-assembly
A_{jet}	area of the jet
A_{poppet}	cross-sectional area at the poppet valve seat sub-assembly
A_{seat}	cross-sectional area of the clapper valve seat opening
A_{valve}	opening area of the main valve
c_{clp}	damping coefficient for the clapper valve
c_{dph}	damping coefficient for the diaphragm system
c_{pop}	damping coefficient for the poppet valve
C_d	coefficient of discharge for main valve flow
C'_d	coefficient of discharge for jet, clapper and poppet valve flow
d_c	diameter of part of the poppet cone which restricts the flow through poppet valve seat opening
d_{jet}	diameter of jet
d_1	diameter of clapper valve front face
d_2	diameter of clapper valve seat opening
d_3	diameter of poppet valve seat opening
F_{fm}	fluid force acting on the membrane
F_m	total force acting on the main diaphragm
h	aircraft altitude
h_{ref}	reference altitude up to which all the parameters are calculated
$\bar{h}$	specific enthalpy
k_{clp}	stiffness value of the clapper-spring
k_{pop}	stiffness value of the poppet-spring
k_{sp}	stiffness value of the spring attached to diaphragm
m_{clp}	mass of the clapper
m_{dph}	mass of the diaphragm
m_{pop}	mass of the poppet

$\dot{m}_c$	rate of change of air-mass within the cabin
$\dot{m}_{cc}$	rate of change of air-mass within the control chamber
$\dot{m}_{clapper}$	mass flow rate of air from control chamber to atmosphere through clapper valve
$\dot{m}_{in}$	mass flow rate of air coming into the cabin
$\dot{m}_{jet}$	mass flow rate of air from cabin to control chamber through jet
$\dot{m}_{leak}$	air mass leakage rate from the valve assembly
$\dot{m}_{out}$	mass flow rate of air from cabin to atmosphere through CPCV
$\dot{m}_{poppet}$	mass flow rate of air from control chamber to atmosphere through poppet valve
$\dot{m}_{valve}$	combined mass flow rate of air from control chamber to atmosphere
P_a	ambient pressure
P_c	cabin pressure
P_{cc}	control chamber pressure
r	inner radius of the diaphragm
R	outer radius of the diaphragm
s	specific entropy
T	temperature
v	throat velocity
V	velocity of ascent/descent of aircraft
x_{abs}	displacement of the absolute pressure capsule
$x_{clapper}$	gap between clapper valve and its valve seat
x_{diff}	displacement of the differential pressure capsule
x_{poppet}	gap between poppet valve and its valve seat
y	vertical lift of the main membrane/diaphragm
α	constant of proportionality in gradient descent algorithm
ΔP_{ca}	pressure difference between cabin and ambient
γ	isentropic index
ν	specific volume
ω_n	natural frequency of the spring-mass-damper system
ρ_c	density of air inside the cabin
ρ_{cc}	density of air inside the control chamber
ζ	damping coefficient of the spring-mass-damper system