ME Annual Research Symposium (MARS) – 2026

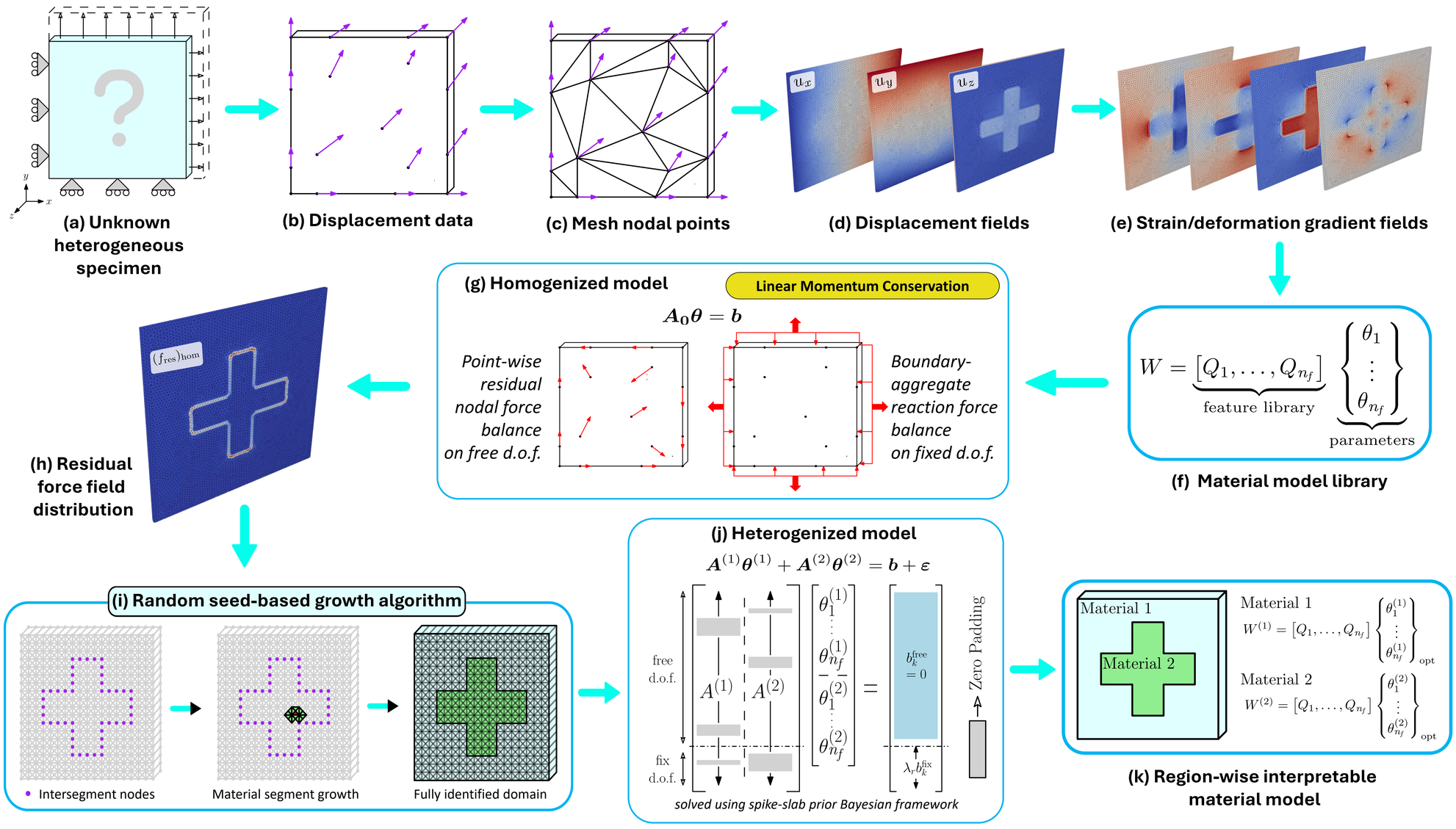
Hetero-EUCLID: Interpretable model discovery for heterogeneous hyperelastic materials using stress-unsupervised learning

Kanhaiya Lal Chaurasiya

Research Advisor: Prof. Akshay Joshi

Department of Mechanical Engineering, Indian Institute of Science

Hetero-EUCLID Framework



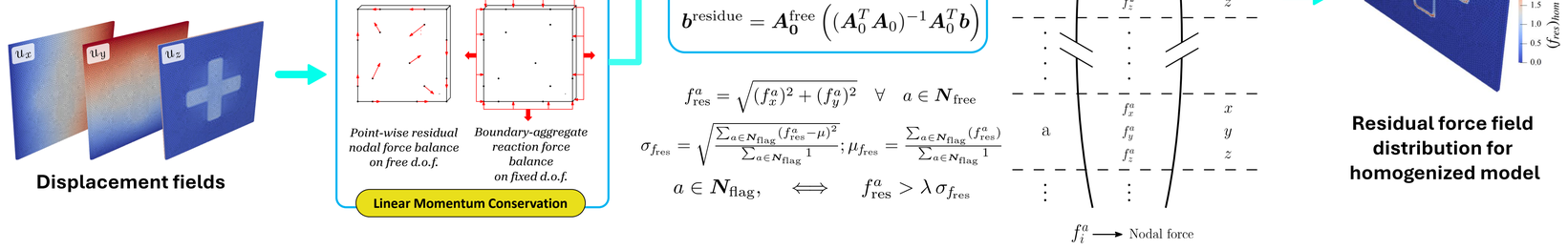
Article DOI access

GitHub open-source code

**Hetero-EUCLID : Heterogeneous hyperelastic - Efficient Unsupervised Constitutive Law Identification and Discovery*

- Framework to segment and model heterogeneous hyperelastic materials interpretably.
- Residual force-based segmentation for accurate material interface detection.
- Bayesian-EUCLID enables sparse, interpretable region-wise material model discovery.
- Robust to displacement noise, non-native mesh, and complex heterogeneities.

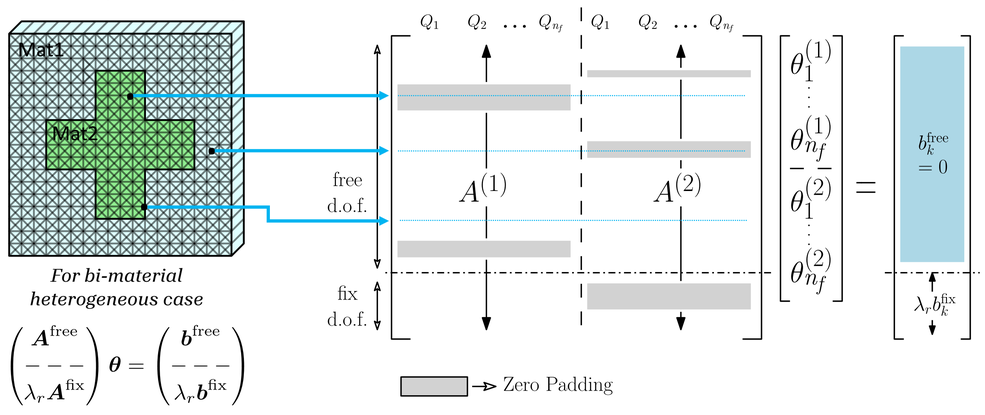
Segmentation



Heterogeneous Hyperelastic Material

Homogeneous Solids: $A\theta = b$

Multi region heterogeneous Solids: $A^{(1)}\theta^{(1)} + A^{(2)}\theta^{(2)} + \cdots + A^{(n_c)}\theta^{(n_c)} = b$



Bayesian-EUCLID | Solving Inverse Problem

Joint minimization: $\arg\min_\theta \left\|A^{\text{free}}\theta\right\|^2 + \lambda_r \left\|A^{\text{fix}}\theta - b^{\text{fix}}\right\|^2 + \lambda_p \|\theta\|_p^p$

Zero internal forces | Reaction force balance | Sparsity regularization

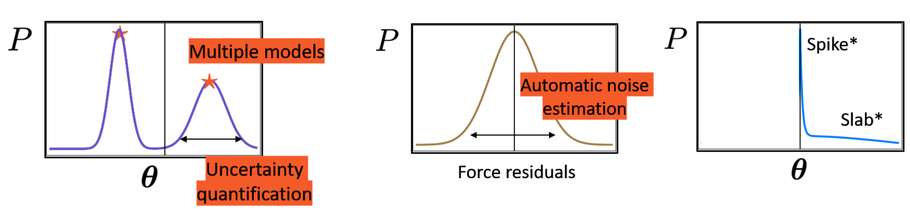
Bayes' rule:

$P(\text{model} \mid \text{eqb., disp.}) \propto P(\text{eqb.} \mid \text{model, disp.}) \times P(\text{model})$

Posterior: material model given observed equilibrium and displacements

Likelihood: equilibrium given material model and displacements

Prior: material model with sparse features



Feature library and models

Model coefficient	Library feature	Model coefficient	Library feature	Model coefficient	Library feature
θ_1	$(\tilde{I}_1 - 3)$	θ_8	$(\tilde{I}_1 - 3)(\tilde{I}_2 - 3)^2$	θ_{15}	$(\tilde{I}_a - 1)^2$
θ_2	$(\tilde{I}_2 - 3)$	θ_9	$(\tilde{I}_2 - 3)^3$	θ_{16}	$(\tilde{I}_a - 1)^3$
θ_3	$(\tilde{I}_1 - 3)^2$	θ_{10}	$(\tilde{I}_1 - 3)^4$	θ_{17}	$(\tilde{I}_a - 1)^4$
θ_4	$(\tilde{I}_1 - 3)(\tilde{I}_2 - 3)$	θ_{11}	$(\tilde{I}_1 - 3)^3(\tilde{I}_2 - 3)$	θ_{18}	$0.5(J^2 - 1) - \ln(J)$
θ_5	$(\tilde{I}_2 - 3)^2$	θ_{12}	$(\tilde{I}_1 - 3)^2(\tilde{I}_2 - 3)^2$	θ_{19}	$(J - 1)^2$
θ_6	$(\tilde{I}_1 - 3)^3$	θ_{13}	$(\tilde{I}_1 - 3)(\tilde{I}_2 - 3)^3$	–	–
θ_7	$(\tilde{I}_1 - 3)^2(\tilde{I}_2 - 3)$	θ_{14}	$(\tilde{I}_2 - 3)^4$	–	–

1	**Neo-Hookean model (NH2)**	$W_{\text{NH2}}(\boldsymbol{C};\boldsymbol{\theta}) = \theta_1(\tilde{I}_1 - 3) + \theta_{19}(J-1)^2$
2	**Isihara model (ISH)**	$W_{\text{ISH}}(\boldsymbol{C};\boldsymbol{\theta}) = \theta_1(\tilde{I}_1 - 3) + \theta_2(\tilde{I}_2 - 3) + \theta_3(\tilde{I}_1 - 3)^2 + \theta_{19}(J-1)^2$
3	**Haines-Wilson model (HW)**	$W_{\text{HW}}(\boldsymbol{C};\boldsymbol{\theta}) = \theta_1(\tilde{I}_1 - 3) + \theta_2(\tilde{I}_2 - 3) + \theta_4(\tilde{I}_1 - 3)(\tilde{I}_2 - 3) + \theta_5(\tilde{I}_1 - 3)^3 + \theta_{19}(J-1)^2$
4	**Ogden model (OG)**	$W_{\text{OG}}(\boldsymbol{C};\boldsymbol{\theta}) = \frac{2\mu}{\eta}\left(\bar{\lambda}_1^\eta + \bar{\lambda}_2^\eta + \bar{\lambda}_3^\eta - 3\right) + \theta_{19}(J-1)^2$ where $\bar{\lambda}_1, \bar{\lambda}_2, \bar{\lambda}_3$ are principal stretches with $\bar{\lambda}_k = J^{-1/3}\lambda_k$
5	**Holzapfel-Gasser-Ogden model (HGO)**	$W_{\text{HGO}}(\boldsymbol{C};\boldsymbol{\theta}) = \theta_1(\tilde{I}_1 - 3) + \theta_{18}\left[\frac{(J^2-1)}{2} - \ln(J)\right] + \frac{k_1}{2k_2}\left[e^{k_2(\tilde{I}_a - 1)^2} - 1\right]$ where $\tilde{I}_a = J^{-2/3}(\boldsymbol{a}\cdot\boldsymbol{C}\boldsymbol{a})$ denotes anisotropic invariant with one fiber family having orientation angle α with $\boldsymbol{a} = (\cos\alpha, \sin\alpha, 0)^T$

Results

Cross-shaped inclusion

$W_{\text{NH2}}(\boldsymbol{C};\boldsymbol{\theta}) = \theta_1(\tilde{I}_1 - 3) + \theta_{19}(J-1)^2$

Zero Noise in Displacement

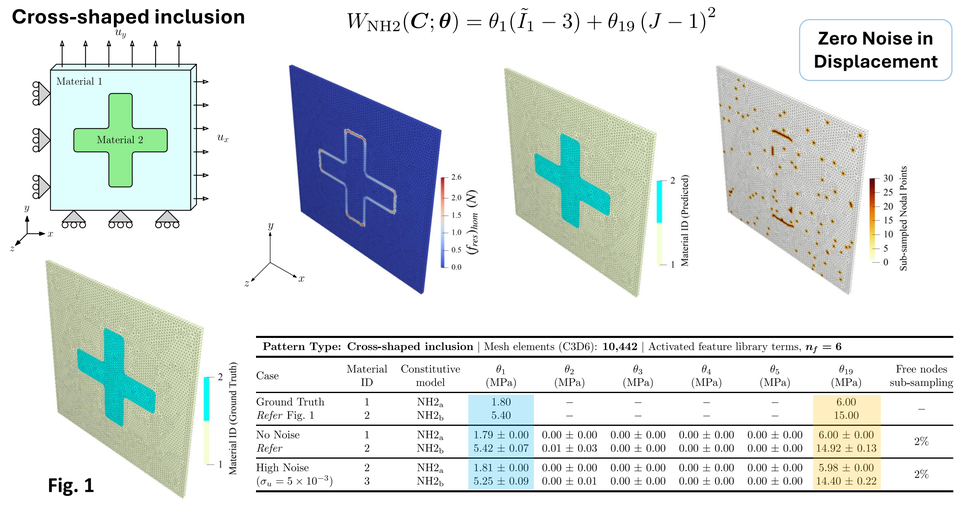


Fig. 1

Pattern Type: Cross-shaped inclusion | Mesh elements (C3D6): **10,442** | Activated feature library terms, $n_f = 6$

Case	Material ID	Constitutive model	θ_1 (MPa)	θ_2 (MPa)	θ_3 (MPa)	θ_4 (MPa)	θ_5 (MPa)	θ_{19} (MPa)	Free nodes sub-sampling
Ground Truth	1	$NH2_a$	1.80	–	–	–	–	6.00	–
Refer Fig. 1	2	$NH2_b$	5.40	–	–	–	–	15.00	
No Noise	1	$NH2_a$	1.79 ± 0.00	0.00 ± 0.00	0.00 ± 0.00	0.00 ± 0.00	0.00 ± 0.00	6.00 ± 0.00	2%
Refer	2	$NH2_b$	5.42 ± 0.07	0.01 ± 0.03	0.00 ± 0.00	0.00 ± 0.00	0.00 ± 0.00	14.92 ± 0.13	
High Noise	2	$NH2_a$	1.81 ± 0.00	0.00 ± 0.00	0.00 ± 0.00	0.00 ± 0.00	0.00 ± 0.00	5.98 ± 0.00	2%
($\sigma_u = 5 \times 10^{-3}$)	3	$NH2_b$	5.25 ± 0.09	0.00 ± 0.01	0.00 ± 0.00	0.00 ± 0.00	0.00 ± 0.00	14.40 ± 0.22	

Cahn-Hilliard pattern

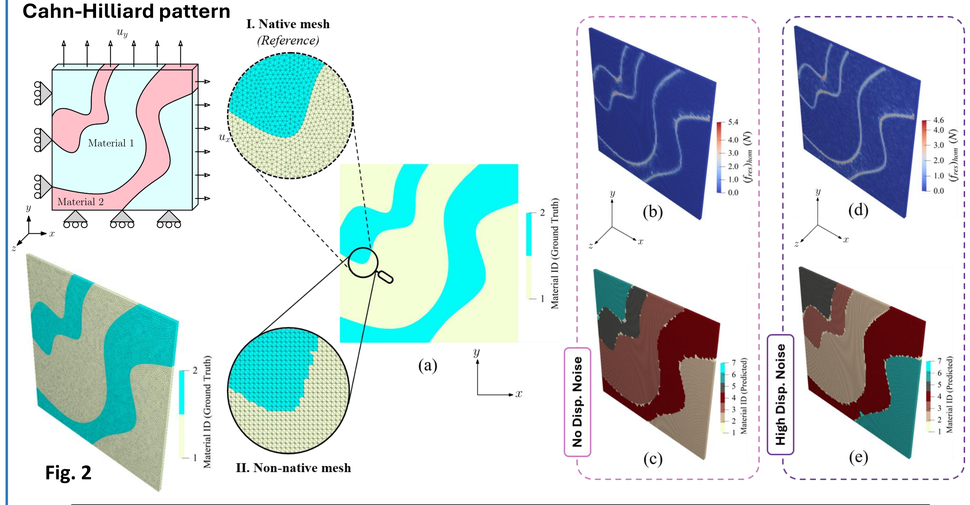


Fig. 2

Pattern Type: Cahn-Hilliard | *Non-native* Mesh elements (C3D6): **19,602** | Activated feature library terms, $n_f = 6$

Case	Material ID	Constitutive model	θ_1 (MPa)	θ_2 (MPa)	θ_3 (MPa)	θ_4 (MPa)	θ_5 (MPa)	θ_{19} (MPa)	Free nodes sub-sampling
Ground Truth, 2(a)	1	$NH2_a$	5.40	–	–	–	–	15.00	–
	2	$NH2_b$	1.80	–	–	–	–	6.00	
No Noise ($\sigma_u = 0$)	6	$NH2_a$	6.05 ± 0.96	0.12 ± 0.43	0.01 ± 0.01	0.00 ± 0.01	0.00 ± 0.00	17.00 ± 1.62	
	5	$NH2_b$	1.29 ± 0.09	0.00 ± 0.00	0.00 ± 0.00	0.00 ± 0.00	0.00 ± 0.00	4.12 ± 0.27	
Bayesian framework	3	$NH2_a$	4.97 ± 0.99	0.12 ± 0.08	0.01 ± 0.03	0.00 ± 0.00	0.00 ± 0.00	14.19 ± 2.50	10%
	4	$NH2_b$	1.77 ± 0.01	0.00 ± 0.00	0.00 ± 0.00	0.00 ± 0.00	0.00 ± 0.00	5.61 ± 0.03	
Refer 2(c)	2	$NH2_a$	5.89 ± 0.09	0.00 ± 0.00	0.00 ± 0.00	0.00 ± 0.02	0.00 ± 0.00	15.25 ± 0.04	
High Noise ($\sigma_u = 5 \times 10^{-3}$)	5	$NH2_a$	7.69 ± 0.07	0.00 ± 0.00	0.04 ± 0.04	0.00 ± 0.00	0.07 ± 0.01	18.82 ± 0.16	
	3	$NH2_b$	1.55 ± 0.04	0.12 ± 0.01	0.00 ± 0.00	0.03 ± 0.00	0.00 ± 0.00	3.49 ± 0.04	
Bayesian framework	2	$NH2_a$	3.36 ± 0.11	0.42 ± 0.06	0.26 ± 0.03	0.01 ± 0.02	0.00 ± 0.00	10.27 ± 0.10	10%
	4	$NH2_b$	1.74 ± 0.02	0.00 ± 0.01	0.00 ± 0.00	0.04 ± 0.00	0.00 ± 0.00	3.46 ± 0.04	
Refer 2(e)	6	$NH2_a$	5.66 ± 0.07	0.00 ± 0.00	0.13 ± 0.04	0.00 ± 0.00	0.00 ± 0.00	13.21 ± 0.09	

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[1] Joshi, A., et al. "Bayesian-EUCLID: Discovering hyperelastic material laws with uncertainties." *Computer Methods in Applied Mechanics and Engineering* 398 (2022): 115225.

[2] Shi, R., et al. "Deep learning without stress data on the discovery of multi-regional hyperelastic properties." *Computational Mechanics* 76.1 (2025): 117-146.

[3] Chaurasiya, K. L., et al. "Hetero-EUCLID: Interpretable model discovery for heterogeneous hyperelastic materials using stress-unsupervised learning." *Computer Methods in Applied Mechanics and Engineering* 452 (2026): 118729.

Acknowledgements

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Dynamics | Data | Design

https://d3lab-iisc.github.io/